* ~~还要讨论drain和undrain的情况~~
* ~~Discussion boundary conditions: 第一类boundary conditions, 第二类boundary conditions~~
* ~~现在只是分析了在温度变化下的第一类loading mode的情况，还有stress变化下的loading mode的情况 – 即讨论superposition的情况~~
* ~~是不是还要讨论下，transportation equation只有一项还是两项的情况~~
* ~~分清楚传热方程式convection还是conduction~~

4. The problem description and poro thermoelastic solution for the high temperature wellbore cementing

By setting up these stress and displacement boundary conditions, we are allow to start our analysis.

Small time solution (undrained cases)

Under the assumption of the plain-strain problem, the formation that the cement is contacted with is characterized by in-situ stress Sx, Sy, formation pressure at borehole wall, borehole wall displacement , formation temperature  and virgin pore pressure.

So firstly, the analysis is starting with some important assumptions. Since usually the length of cementing much longer than the diameter of the wellbore, thus, The generalized plane strain assumption is appropriate analysis this problem by assuming that the geomechanics are characterized by geometries in which boundary conditions are constant along the direction of the infinitely long borehole axis. In line with the loading decomposition scheme proposed by detournay & cheng (ref here), this porothermoelastic problem could be decomposed into three fundamental loading modes for better physical interpretations: (1) far-field isotropic stress; (2) temperature load; (3) virgin pore pressure. Finally, the principle of superposition is used to obtain the complete solutions due the linearity nature of this problem.

In the interest of this paper, taking permanently plugging in the P&A system as an example, this paper is particularly interested in two scenarios: placing the cement plug into soft formation and hard formation. The main difference between these two cases is that the deformation of the cement plug (here mainly refers to radial deformation under the plain-strain problem). The transition and large time solution for a thermoporoelastic cylinder subjected to mechanical, hydraulic and thermal loading has been derived, and all the results will be presented and discussed in the next section. However, it is first useful to derive the small time solution, that is, the undrained behaviors, corresponding to each of three modes of loadings. Under the scope of content of this article, the induced pore pressure under three different modes is in particular interest and will discuss below.

According to the constitute equation,

 where 

Under this loading mode 1, immediately after load is applied, at , the stress state is uniform with (where ). Hence and the early-time (undrained) case is obtained by enabling  from the constitutive equation (ref), which will obtain the expressions of  at :



It is noticeable that the first term on the right-hand sides of the equation (ref) is due to poroelastic response, with the second term corresponding to the effect of thermal filtration coupling. But it is important to realize that the second term from the equation (ref) that arise from thermal filtration coupling typically falls into the range of . Hence, under the undrained conditions, the effect of the thermal infiltration coupling on the hydraulic pore pressure is not significant. At large time, the transient pore pressure fields have dissipated so .

Similarly, the early-time (undrained) case under the loading mode 2 is showing same behavior of the induced pore pressure. By characterizing  and from the constitutive equation (ref), theat will be obtained as:



Again, under the undrained conditions, the thermal filtration part of the induced pore pressure is still small in magnitude of , which can be considered as negligible. At larger time, the same uniform state of stress is restored, but now with 

Under mode 3 loading, all the stress conditions are given as zero at small time with , so that the undrained solution is characterized by all quantities equal to zero. That is, there is no jump in the solution from the initial conditions at . At large time, the thermal induced pore pressure has already dissipated whereand the temperature field is uniform as 

Under such conditions, the temperature and pore pressure boundary conditions is set up as the Dirichlet Boundary condition.